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Local versus global interactions in nonequilibrium transitions: A model of social dynamics

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A nonequilibrium system of locally interacting elements in a lattice with an absorbing order-disorder phase transition is studied under the effect of additional interacting fields. These fields are shown to produce interesting effects in the collective behavior of this system. Both for autonomous and external fields, disorder grows in the system when the probability of the elements to interact with the field is increased. There exists a threshold value of this probability beyond which the system is always disordered. The domain of parameters of the ordered regime is larger for nonuniform local fields than for spatially uniform fields. However, the zero field limit is discontinous. In the limit of vanishingly small probability of interaction with the field, autonomous or external fields are able to order a system that would fall in a disordered phase under local interactions of the elements alone. We consider different types of fields which are interpreted as forms of mass media acting on a social system in the context of Axelrod's model for cultural dissemination.

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I. INTRODUCTION

The emergence of nontrivial collective behavior in spatiotemporal dynamical systems is a central issue in the current research on complex systems, as in many physical, chemical, biological, economic, and social phenomena. There are a variety of processes occurring in these systems where both spatially local and global interactions extending all over the system coexist and contribute in different and competing ways to the collective dynamics. Some examples include Turing patterns [1] (with slow and fast diffusion), Ginzburg-Landau dynamics [2], surface chemical reactions [3], sand dunes (with the motions of wind and of sand) [4], and pattern formation in some biological systems [5]. Recently, the collective behavior of dynamical elements subject to both local and global interactions has been experimentally investigated in arrays of chaotic electrochemical cells [6]. Many of these systems can be modeled as networks of coupled dynamical units with coexisting local and global interactions [7]. Similarly, the phenomena of pattern formation and collective behavior induced by external forcing on spatiotemporal systems, such as chemical reactions [8,9] or granular media [10], has also been considered. The analogy between external forcing and global coupling in spatiotemporal dynamical systems has recently been explored in the framework of coupled map lattice models [11,12]. It has been found that, under some circumstances, the collective behavior of an autonomous spatiotemporal system with local and global interactions is equivalent to that of a driven spatiotemporal system possessing similar local couplings as in the autonomous system.

The addition of a global interaction to a locally coupled system is known to be able to induce phenomena not present in that system, such as chaotic synchronization and new spatial patterns. However, the classification and description of generic effects produced by external fields or global coupling in a nonequilibrium system of locally interacting units is still an open general question. The common wisdom for equilibrium systems is that under a strong external field, local interactions become negligible, and the system orders following the external field. For nonequilibrium nonpotential dynamics [13] this is not necessarily the case, and nontrivial effects might arise depending on the dynamical rules.

This problem is, in particular, relevant for recent studies of social phenomena in the general framework of complex systems. The aim is to understand how collective behaviors arise in social systems. Several mathematical models, many of them based on discrete-time and discrete-space dynamical systems, have been proposed to describe a variety of phenomena occurring in social dynamics [14–22]. In this context, specially interesting is the lattice model introduced by Axelrod [23] to investigate the dissemination of culture among interacting agents in a society [22,24–30]. The state of an agent in this model is described by a set of individual cultural features. The local interaction between neighboring agents depends on the cultural similarities that they share and similarity is enhanced as a result of the interaction. From the point of view of statistical physics, this model is appealing because it exhibits a nontrivial out of equilibrium transition between an ordered phase (a homogeneous culture) and a disordered (multicultural) one, as in other well studied lattice systems with phase ordering properties [31]. The additional effect of global coupling in this system has been considered as a model of influence of mass media [24]. It has also been shown that the addition of external influences, such as random perturbations [28] or a fixed field [32], can induce new order-disorder nonequilibrium transitions in the collective behavior of Axelrod's model. However, a global picture of the results of the competition between the local interaction among the agents and the interaction through a global coupling field or an external field is missing. In this paper we address this general question in the specific context of Axelrod's model.

We deal with states of the elements of the system and interacting fields described by vectors whose components can take discrete values. The interaction dynamics of the elements among themselves and with the fields is based on the similarity between state vectors, defined as the fraction of components that these vectors have in common. We consider interaction fields that originate either externally (an external forcing) or from the contribution of a set of elements in the system (an autonomous dynamics) such as global or partial coupling functions. Our study allows to compare the effects that driving fields or autonomous fields of interaction have on the collective properties of systems with this type of nonequilibrium dynamics. In the context of social phenomena, our scheme can be considered as a model for a social system interacting with global or local mass media that represent endogenous cultural influences or information feedback, as well as a model for a social system subject to an external cultural influence. A usual equilibrium idea is that the application of a field should enhance order in a system. Our results indicate that here this is not the case. On the contrary, disorder builds up by increasing the probability of interaction of the elements with the field. This occurs independently of the nature (either external or autonomous) of the field of interaction added to the system. Moreover, we find that a spatially nonuniform field of interaction may actually produce less disorder in the system than a uniform field.

The model, including the description of three types of interaction fields being considered, is presented in Sec. II. In Sec. III, the effects of the fields in the ordered phase of the system are shown, while Sec. IV analyzes these effects in the disordered phase. Section V contains a global picture and interpretation of our results.

II. THE MODEL

The system consists of N elements as the sites of a square lattice. The state c_i of element i is defined as a vector of F components $\sigma_i=(\sigma_{i1},\sigma_{i2},\ldots,\sigma_{iF})$. In Axelrod's model, the F components of c_i correspond to the cultural features describing the F-dimensional culture of element i. Each component σ_{if} can take any of the q values in the set $\{0,1,\ldots,q-1\}$ (called cultural traits in Axelrod's model). As an initial condition, each element is randomly and independently assigned one of the q^F state vectors with uniform probability. We introduce a vector field M with components $(\mu_{i1},\mu_{i2},\ldots,\mu_{iF})$. Formally, we treat the field at each element i as an additional neighbor of i with whom an interaction is possible. The field is represented as an additional element $\phi(i)$ such that $\sigma_{\phi(i)f}=\mu_{if}$ in the definition given below of the dynamics. The strength of the field is given by a constant parameter $B\in[0,1]$ that measures the probability of interaction with the field. The system evolves by iterating the following steps:

(1) Select at random an element i on the lattice (called active element).

(2) Select the source of interaction j. With probability B set $j=\phi(i)$ as an interaction with the field. Otherwise, choose element j at random among the four nearest neighbors (the von Neumann neighborhood) of i on the lattice.

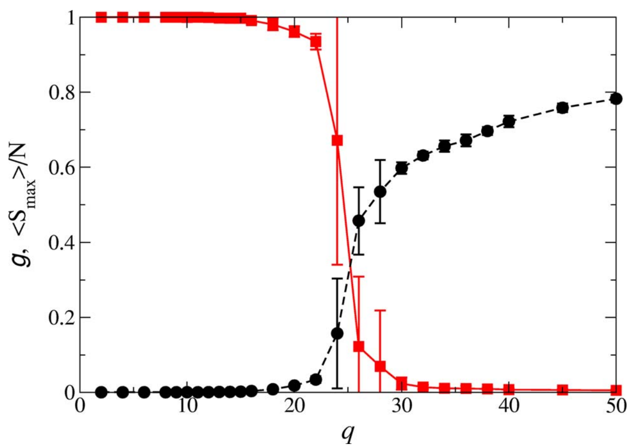

FIG. 1. (Color online) Order parameters g (circles) and $\langle S_{\max}\rangle/N$ (squares) as a function of q, in the absence of a field $B=0$.

(3) Calculate the overlap (number of shared components) $l(i,j)=\sum_{f=1}^{F}\delta_{\sigma_{if},\sigma_{jf}}$. If $0<l(i,j)<F$, sites i and j interact with probability $l(i,j)/F$. In case of interaction, choose h randomly such that $\sigma_{ih}\neq\sigma_{jh}$ and set $\sigma_{ih}=\sigma_{jh}$.

(4) Update the field M if required (see definitions of fields below). Resume at (1).

Step (3) specifies the basic rule of a nonequilibrium dynamics which is at the basis of most of our results. It has two ingredients: (i) a similarity rule for the probability of interaction, and (ii) a mechanism of convergence to a homogeneous state.

Before considering the effects of the field M, let us review the original model without field ($B=0$). In any finite network the dynamics settles into an absorbing state, characterized by either $l(i,j)=0$ or $l(i,j)=F$, for all pairs of neighbors (i,j). Homogeneous ("monocultural") states correspond to $l(i,j)=F$, $\forall i,j$, and obviously there are q^F possible configurations of this state. Inhomogeneous (multicultural) states consist of two or more homogeneous domains interconnected by elements with zero overlap and therefore with frozen dynamics. A domain is a set of contiguous sites with identical state vectors. It has been shown that the system reaches ordered, homogeneous states for $q<q_c$ and disordered, inhomogeneous states for $q>q_c$, where q_c is a critical value that depends on F [25–29]. This order-disorder nonequilibrium transition is of second order in one-dimensional systems and of first order in two-dimensional systems [30]. It has also been shown that the inhomogeneous configurations are not stable: single feature perturbations acting on these configurations unfreeze the dynamics. Under repeated action of these perturbations the system reaches an homogeneous state [28].

To characterize the transition from a homogeneous state to a disordered state, we consider as an order parameter the average fraction of cultural domains $g=\langle N_g\rangle/N$. Here N_g is the number of domains formed in the final state of the system for a given realization of initial conditions. Figure 1 shows the quantity g as a function of the number of options per component q, for $F=5$, when no field acts on the system ($B=0$). For values of $q<q_c\approx 25$, the system always reaches a homogeneous state characterized by values $g\to 0$. On the

other hand, for values of $q>q_c$, the system settles into a disordered state, for which $\langle N_g\rangle \gg 1$. Another previously used order parameter [25,27], the average size of the largest domain size, $\langle S_{\max}\rangle/N$, is also shown in Fig. 1 for comparison. In this case, the ordered phase corresponds to $\langle S_{\max}\rangle/N=1$, while complete disorder is given by $\langle S_{\max}\rangle/N\to 0$. Unless otherwise stated, our numerical results throughout the paper are based on averages over 50 realizations for systems of size $N=40\times 40$, and $F=5$.

Let us now consider the case where the elements on the lattice have a nonzero probability to interact with the field ($B>0$). We distinguish three types of fields.

(i) The *external field* is spatially uniform and constant in time. Initially for each component f, a value $\epsilon_f \in \{1,\ldots,q\}$ is drawn at random and $\mu_{if}=\epsilon_f$ is set for all elements i and all components f. It corresponds to a constant, external driving field acting uniformly on the system. A constant external field can be interpreted as a specific cultural state (such as advertising or propaganda) being imposed by controlled mass media on all the elements of a social system [32].

(ii) The *global field* is spatially uniform and may vary in time. Here μ_{if} is assigned the most abundant value exhibited by the fth component of all the state vectors in the system. If the maximally abundant value is not unique, one of the possibilities is chosen at random with equal probability. This type of field is a global coupling function of all the elements in the system. It provides the same global information feedback to each element at any given time but its components may change as the system evolves. In the context of cultural models [24], this field may represent a global mass media influence shared identically by all the agents and which contains the most predominant trait in each cultural feature present in a society (a "global cultural trend").

(iii) The *local field*, is spatially nonuniform and nonconstant. Each component μ_{if} is assigned the most frequent value present in component f of the state vectors of the elements belonging to the von Neumann neighborhood of element i. If there are two or more maximally abundant values of component f one of these is chosen at random with equal probability. The local field can be interpreted as local mass media conveying the "local cultural trend" of its neighborhood to each element in a social system.

Case (i) corresponds to a driven spatiotemporal dynamical system. On the other hand, cases (ii) and (iii) can be regarded as autonomous spatiotemporal dynamical systems. In particular, a system subject to a global field corresponds to a network of dynamical elements possessing both local and global interactions. Both the constant external field and the global field are uniform. The local field is spatially nonuniform; it depends on the site i. In the context of cultural models, systems subject to either local or global fields describe social systems with endogenous cultural influences, while the case of the external field represents and external cultural influence.

The strength of the coupling to the interaction field is controlled by the parameter B. We shall assume that B is uniform, i.e., the field reaches all the elements with the same probability. In the cultural dynamics analogy, the parameter B can be interpreted as the probability that the mass media vector has to attract the attention of the agents in the social system. The parameter B represents enhancing factors of the mass media influence that can be varied, such as its amplitude, frequency, attractiveness, etc.

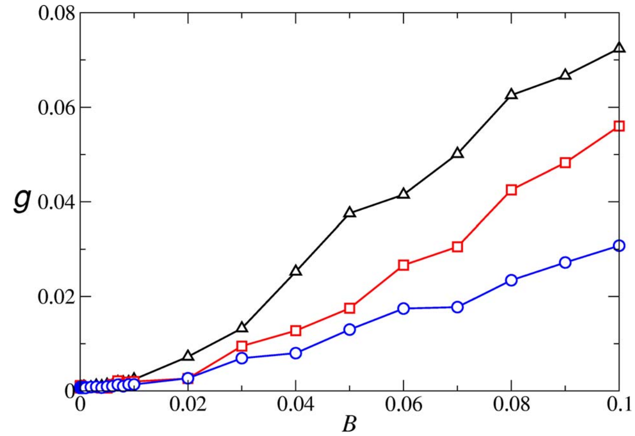

FIG. 2. (Color online) Order parameter g as a function of the coupling strength B of an external (squares), global (circles), and local (triangles) field. Parameter value $q=10<q_c$.

III. EFFECTS OF AN INTERACTING FIELD FOR $q<q_c$

In the absence of any interaction field, the system settles into one of the possible q^F homogeneous states for $q<q_c$ (see Fig. 1). Figure 2 shows the order parameter g as a function of the coupling strength B for the three types of fields. When the probability B is small enough, the system still reaches in its evolution a homogeneous state ($g\to 0$) under the action of any of these fields. In the case of an external field, the homogeneous state reached by the system is equal to the field vector [32]. Thus, for small values of B, a constant external field imposes its state over all the elements in the system, as one may expect. With a global or with a local field, however, for small B the system can reach any of the possible q^F homogeneous states, depending on the initial conditions. Regardless of the type of field, there is a transition at a threshold value of the probability B_c from a homogeneous state to a disordered state characterized by an increasing number of domains as B is increased. Thus, we find the counterintuitive result that, above some threshold value of the probability of interaction, a field induces disorder in a situation in which the system would order (homogeneous state) under the effect alone of local interactions among the elements.

The threshold values of the probability B_c for each type of field, obtained by a regression fitting [32], are plotted as a function of q in the phase diagram of Fig. 3. The threshold value B_c for each field decreases with increasing q for $q<q_c$. The value $B_c=0$ for the three fields is reached at $q=q_c\approx 25$, corresponding to the critical value in absence of interaction fields observed in Fig. 1. For each case, the threshold curve B_c vs q in Fig. 3 separates the region of disorder from the region where homogeneous states occur on the space of parameters (B,q). For $B>B_c$, the interaction with the field dominates over the local interactions among

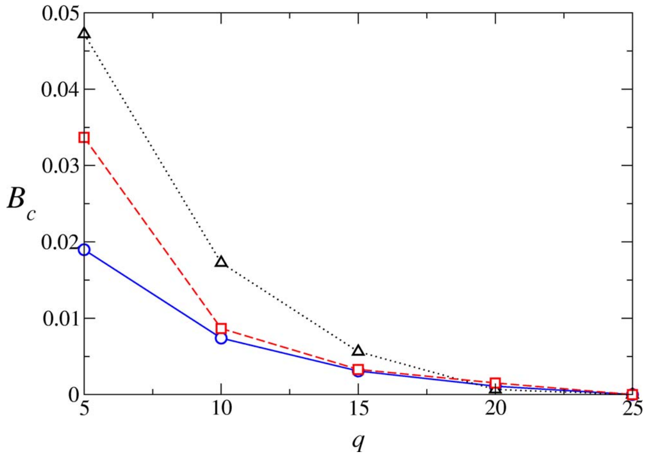

FIG. 3. (Color online) Threshold values B_c for $q<q_c$ corresponding to the different fields. Each line separates the region of order (above the line) from the region of disorder (below the line) for an external (squares), global (circles), and local (triangles) field.

the individual elements in the system. Consequently, elements whose states exhibit a greater overlap with the state of the field have more probability to converge to that state. This process contributes to the differentiation of states between neighboring elements and to the formation of multiple domains in the system for large enough values of the probability B.

Note that the region of homogeneous ordered states in the (B,q) space in Fig. 3 is larger for the local field than for the external and the global fields. A nonuniform field provides different influences on the agents, while the interaction with uniform fields is shared by all the elements in the system. The local field (spatially nonuniform) is less efficient than uniform fields in promoting the formation of multiple domains, and therefore order is maintained for a larger range of values of B when interacting with a local field.

IV. EFFECTS OF AN INTERACTING FIELD FOR $q>q_c$

When there are no additional interacting fields ($B=0$), the system always freezes into disordered states for $q>q_c$. Figure 4 shows the order parameter g as a function of the probability B for the three types of fields. The effect of a field for $q>q_c$ depends on the magnitude of B. In the three cases we see that for $B\to 0$, g drops to values below the reference line corresponding to its value when $B=0$. Thus, the limit $B\to 0$ does not recover the behavior of the model with only local nearest-neighbor interactions. The fact that for $B\to 0$ the interaction with a field increases the degree of order in the system is related to the nonstable nature of the inhomogeneous states in Axelrod's model. When the probability of interaction B is very small, the action of a field can be seen as a sufficient perturbation that allows the system to escape from the inhomogeneous states with frozen dynamics. The role of a field in this situation is similar to that of noise applied to the system, in the limit of vanishingly small noise rate [28].

The drop in the value of g as $B\to 0$ from the reference value ($B=0$) that takes place for the local field in Fig. 4 is

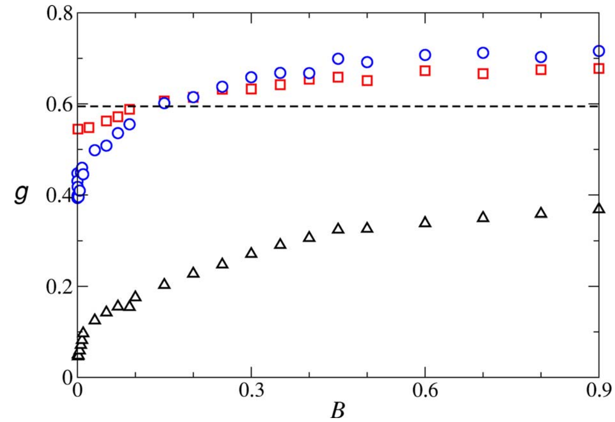

FIG. 4. (Color online) Order parameter g as a function of the coupling strength B of an external (squares), global (circles), and local (triangles) field. The horizontal dashed line indicates the value of g at $B=0$. Parameter value $q=30$.

more pronounced than the corresponding drops for uniform fields. This can be understood in terms of a greater efficiency of a nonuniform field as a perturbation that allows the system to escape from a frozen inhomogeneous configuration. Increasing the value of B results, in all three types of fields, in an enhancement of the degree of disorder in the system, but the local field always keeps the amount of disorder, as measured by g, below the value obtained for $B=0$. Thus a local field has a greater ordering effect than both the global and the external fields for $q>q_c$.

The behavior of the order parameter g for larger values of B can be described by the scaling relation $g\sim B^{\beta}$, where the exponent β depends on the value of q. Figure 5 shows a log-log plot of g as a function of B, for the case of a global field, verifying this relation. This result suggests that g should drop to zero as $B\to 0$. The partial drops observed in Fig. 4 seem to be due to finite size effects for $B\to 0$. A detailed investigation of such finite size effects is reported in Fig. 6 for the case of the global field It is seen that, for very small values of B, the values of g decrease as the system size N increases. However, for values of $B\gtrsim 10^{-2}$, the variation of the size of the system does not affect g significantly.

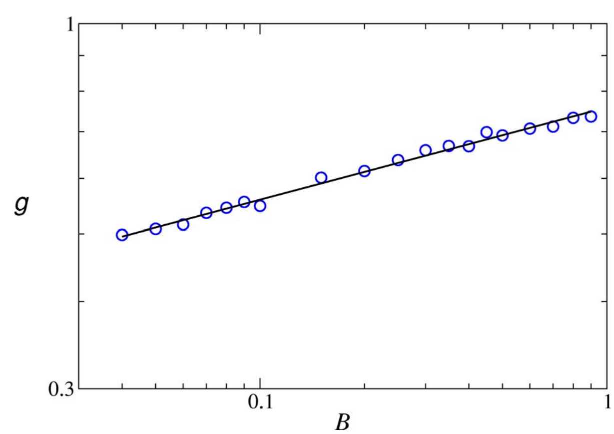

FIG. 5. (Color online) Scaling of the order parameter g with the coupling strength to the global field B. The slope of the fitting straight line is $\beta=0.13\pm0.01$. Parameter value $q=30>q_c$.

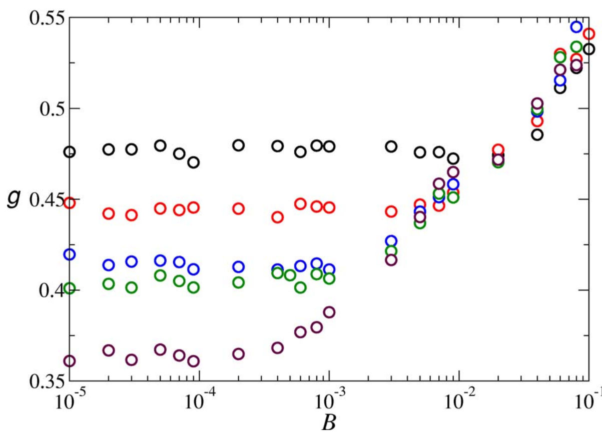

FIG. 6. (Color online) Finite size effects at small values of the strength B of a global field Order parameter g as a function of B is shown for system sizes $N=20^2$, 30^2, 40^2, 50^2, 70^2 (from top to bottom). Parameter value $q=30$.

Figure 7 displays the dependence of g on the size of the system N when $B\to 0$ for the three interaction fields being considered. For each size N, a value of g associated with each field was calculated by averaging over the plateau values shown in Fig. 6 in the interval $B\in[10^{-5},10^{-3}]$. The mean values of g obtained when $B=0$ are also shown for reference. The order parameter g decreases for the three fields as the size of the system increases; in the limit $N\to\infty$ the values of g tend to zero and the system becomes homogeneous in the three cases. For small values of B, the system subject to the local field exhibits the greatest sensitivity to an increase of the system size, while the effect of the constant external field is less dependent on system size. The ordering effect of the interaction with a field as $B\to 0$ becomes more evident for a local (nonuniform) field. But, in any case, the system is driven to full order for $B\to 0$ in the limit of infinite size by any of the interacting fields considered here.

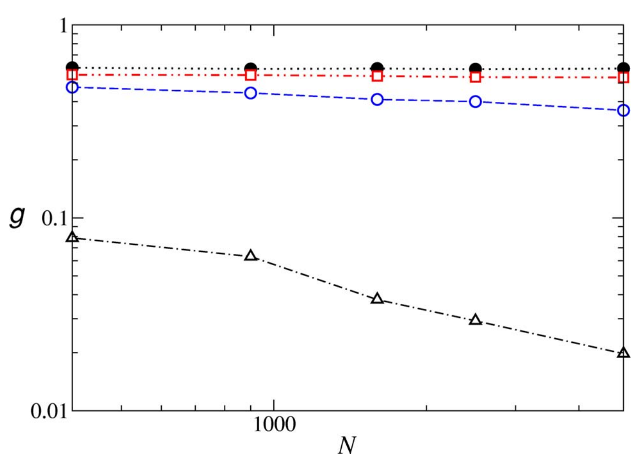

FIG. 7. (Color online) Mean value of the order parameter g as a function of the system size N without field ($B=0$, solid circles), and with an external (squares), global (circles), and local (triangles field). Parameter value $q=30$.

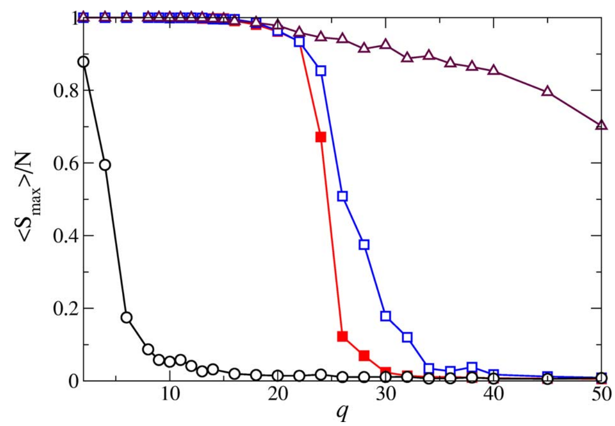

FIG. 8. (Color online) Influence of the interacting field on the nonequilibrium order-disorder transition as described by the order parameter $\langle S_{\max}\rangle/N$. Results are shown for $B=0$ (solid squares), a global [$B=10^{-5}$ (empty squares), $B=0.3$ (circles)] and a local [$B=10^{-5}$ (triangles)] field. Parameter value $F=3$.

V. SUMMARY AND CONCLUSIONS

We have analyzed a nonequilibrium lattice model of locally interacting elements and subject to additional interacting fields. The state variables are described by vectors whose components take discrete values. We have considered the cases of a constant external field, a global field, and a local field. The interaction dynamics, based on the similarity or overlap between vector states, produces several nontrivial effects in the collective behavior of this system. Namely, we find two main effects that contradict intuition based on the effect of interacting fields in equilibrium systems where the dynamics minimizes a potential function. First, we find that an interacting field might disorder the system: For parameter values for which the system orders due to the local interaction among the elements, there is a threshold value B_c of the probability of interaction with a field. For $B>B_c$ the system becomes disordered. This happens because there is a competition between the consequences of the similarity rule applied to the local interactions among elements, and applied to the interaction with the field. This leads to the formation of domains and to a disordered system. A second effect is that, for parameter values for which the dynamics based on the local interaction among the elements leads to a frozen disordered configuration, very weak interacting fields are able to order the system. However, increasing the strength of interaction with the field produces growing disorder in the system. The limit $B\to 0$ is discontinuous and the ordering effect for $B\ll 1$ occurs because the interaction with the field acts as a perturbation on the non stable disordered configurations with frozen dynamics appearing for $B=0$. In this regard, the field behaves similarly to a random fluctuation acting on the system, which always induces order for small values of the noise rate [28].

These results are summarized in Fig. 8 which shows, for different values of B, the behavior of the order parameter $\langle S_{\max}\rangle/N$ previously considered in Fig. 1. For small values of B, the interaction with a field can enhance order in the system: for $q<q_c$ interaction with a field preserves homogene-

ity, while for $q > q_c$ it causes a drop in the degree of disorder in the system. In an effective way the nonequilibrium order-disorder transition is shifted to larger values of q when B is nonzero but very small. For larger values of B the transition shifts to smaller values of q and the system is always disordered in the limiting case $B \to 1$. This limiting behavior is useful to understand the differences with ordinary dynamics leading to thermal equilibrium in which a strong field would order the system. In our nonequilibrium case, the similarity rule of the dynamics excludes the interaction of the field with elements with zero overlap with the field. Since the local interaction among the elements is negligible in this limit, there is no mechanism left to change situations of zero overlap and the system remains disordered. We have calculated, for the three types of field considered, the corresponding boundary in the space of parameters (B, q) that separates the ordered phase from the disordered phase. In the case of a constant external field, the ordered state in this phase diagram always converges to the state prescribed by the constant field vector. The nonuniform local field has a greater ordering effect than the uniform (global and constant external) fields in the regime $q > q_c$. The range of values of B for which the system is ordered for $q < q_c$ is also larger for the nonuniform local field.

In spite of the differences mentioned between uniform and nonuniform fields, it is remarkable that the collective behavior of the system displays analogous phenomenology for the three types of fields considered, although they have different nature. At the local level, they act in the same manner, as a "fifth" effective neighbor whose specific source becomes irrelevant. In particular, both uniform fields, the global coupling and the external field, produce very similar behavior of the system. Recently, it has been found that, under some circumstances, a network of locally coupled dynamical elements subject to either global interactions or to a uniform external drive exhibits the same collective behavior [11,12]. The results from the present nonequilibrium lattice model suggest that collective behaviors emerging in autonomous and in driven spatiotemporal systems can be equivalent in a more general context.

In the context of Axelrod's model for the dissemination of culture [23] the interacting fields that we have considered can be interpreted as different kinds of mass media influences acting on a social system. In this context, our results suggest that both, an externally controlled mass media or mass media that reflect the predominant cultural trends of the environment, have similar collective effects on a social system. We found the surprising result that, when the probability of interacting with the mass media is sufficiently large, mass media actually contribute to cultural diversity in a social system, independently of the nature of the media. Mass media is only efficient in producing cultural homogeneity in conditions of weak broadcast of a message, so that local interactions among individuals can be still effective in constructing some cultural overlap with the mass media message. Local mass media appear to be more effective in promoting uniformity in comparison to global, uniform broadcasts.

Future extensions of this work should include the consideration of noise and complex networks of interaction.

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